

# Package ‘uniformly’

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**Title** Uniform Sampling

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**Description** Uniform sampling on various geometric shapes, such as spheres, ellipsoids, simplices.

**License** GPL-3

**Encoding** UTF-8

**URL** <https://github.com/stla/uniformly>

**BugReports** <https://github.com/stla/uniformly/issues>

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rphong\_on\_hemisphere    *Sampling on hemisphere*

---

## Description

Sampling on a hemisphere according to the Phong density (dimension 3).

## Usage

```
rphong_on_hemisphere(n, alpha = 0, r = 1)
```

## Arguments

n	number of simulations
alpha	parameter of the Phong density, a positive number; 0 for uniform sampling (default)
r	radius

## Value

The simulations in a n times 3 matrix.

**Examples**

```
## Not run:  
library(rgl)  
sims <- rphong_on_hemisphere(400, alpha = 10)  
spheres3d(0, 0, 0, color = "red", alpha = 0.5)  
points3d(sims)  
## End(Not run)
```

---

runif\_cube

*Uniform sampling on/in cube*

---

**Description**

Uniform sampling on or in a cube (arbitrary dimension).

**Usage**

```
runif_in_cube(n, d, O = rep(0, d), r = 1)
```

```
runif_on_cube(n, d, O = rep(0, d), r = 1)
```

**Arguments**

n	number of simulations
d	dimension
O	center of the cube
r	radius (half-side) of the cube

**Value**

The simulations in a n times d matrix.

**Examples**

```
sims <- runif_on_cube(60, d = 2)  
plot(sims, xlim = c(-1,1), ylim = c(-1,1), pch = 19, asp = 1)  
sims <- runif_in_cube(50, d = 3)  
library(scatterplot3d)  
scatterplot3d(sims, pch = 19, highlight.3d = TRUE, asp = 1)
```

---

runif\_ellipsoid      *Uniform sampling on/in ellipsoid*

---

### Description

Uniform sampling on an ellipsoid or in an ellipsoid (arbitrary dimension).

### Usage

```
runif_on_ellipsoid(n, A, r)
```

```
runif_in_ellipsoid(n, A, r)
```

### Arguments

n	number of simulations
A	symmetric positive-definite matrix defining the ellipsoid (see Details)
r	"radius" (see Details)

### Details

The ellipsoid is the set of vectors  $x$  satisfying  $t(x) \%*\% A \%*\% x == r^2$ .

### Value

The simulations in a matrix with  $n$  rows.

### Examples

```
A <- rbind(c(2, 1), c(1, 1))
r <- 2
sims <- runif_on_ellipsoid(30, A, r)
plot(sims, xlim = c(-2, 2), ylim = c(-3, 3), asp = 1, pch = 19)
sims <- runif_in_ellipsoid(100, A, r)
plot(sims, xlim = c(-2, 2), ylim = c(-3, 3), asp = 1, pch = 19)
# 3D example
A <- matrix(c(5,1,1, 1,3,1, 1,1,1), ncol = 3L)
r <- 2
# draw the ellipsoid
library(misc3d)
x <- seq(-1, 1, len = 50)
y <- seq(-1.5, 1.5, len = 50)
z <- seq(-2.7, 2.7, len = 50)
g <- as.matrix(expand.grid(x = x, y = y, z = z))
voxel <-
  array(apply(g, 1L, function(v) t(v) \%*\% A \%*\% v), dim = c(50, 50, 50))
isosurface <- computeContour3d(voxel, max(voxel), r^2, x = x, y = y, z = z)
drawScene.rgl(makeTriangles(isosurface, alpha = 0.3))
# simulate and plot points on ellipsoid
```

```
library(rgl)
sims <- runif_on_ellipsoid(200, A, r)
points3d(sims)
```

---

runif\_in\_annulus      *Uniform sampling in an annulus*

---

**Description**

Uniform sampling in an annulus (dimension 2).

**Usage**

```
runif_in_annulus(n, 0, r1, r2)
```

**Arguments**

n	number of simulations
0	center of the annulus
r1	inner radius
r2	outer radius

**Value**

The simulations in a n times 2 matrix.

**Examples**

```
sims <- runif_in_annulus(100, c(0, 0), 1, 2)
plot(sims, xlim = c(-2, 2), ylim = c(-2, 2), asp = 1, pch = 19)
```

---

runif\_in\_pball      *Uniform sampling in a p-ball*

---

**Description**

Uniform sampling in a p-ball (arbitrary dimension).

**Usage**

```
runif_in_pball(n, d, p, r = 1)
```

**Arguments**

n	number of simulations
d	dimension
p	exponent in the p-norm, a positive number
r	positive number, the radius

**Value**

The simulations in a n times d matrix.

**Examples**

```
sims <- runif_in_pball(500, d = 2, p = 1)
plot(sims, xlim = c(-1, 1), ylim = c(-1, 1), asp = 1)
```

---

runif\_in\_polygon      *Uniform sampling in a polygon*

---

**Description**

Uniform sampling in a polygon (dimension 2).

**Usage**

```
runif_in_polygon(n, vertices, center = "centroid")
```

**Arguments**

n	number of simulations
vertices	two-columns matrix giving the vertices (rows); the vertices must be ordered (clockwise or counterclockwise)
center	a point with respect to which the polygon is star-shaped, or "centroid" (default) to take the centroid (see Details)

**Details**

This function works for a star-shaped polygon, that is, a polygon that contains a point from which the entire polygon boundary is visible. This point must be given in the center argument. If the polygon is convex, any point inside the polygon is suitable (thus the default option of the center argument is appropriate in this case).

**Value**

The simulations in a n times 2 matrix.

**Examples**

```
vs <- matrix(c(0.951056516295154, 0.309016994374947,
              0.224513988289793, 0.309016994374947,
              -0.951056516295154, 0.309016994374948,
              -0.363271264002681, -0.118033988749895,
              0.587785252292473, -0.809016994374948,
              0.36327126400268, -0.118033988749895,
              0, 1,
              -0.224513988289793, 0.309016994374947,
              -0.587785252292473, -0.809016994374947,
              0, -0.381966011250105),
            ncol=2, byrow=TRUE)
sims <- runif_in_polygon(500, vs)
plot(sims, xlim = c(-1, 1), ylim = c(-1, 1), pch = 19, asp = 1)
```

---

runif_in_simplex	<i>Uniform sampling in a simplex</i>
------------------	--------------------------------------

---

**Description**

Uniform sampling in a simplex (arbitrary dimension).

**Usage**

```
runif_in_simplex(n, simplex)
```

**Arguments**

n	number of simulations
simplex	a (d+1) times d matrix giving the vertices of the simplex (rows)

**Value**

The simulations in a n times d matrix.

**Note**

In dimension 3, you can use [runif\\_in\\_tetrahedron](#) instead.

**Examples**

```
simplex <- rbind(c(0,0,0), c(1,0,0), c(1,1,0), c(1,1,2))
sims <- runif_in_simplex(1000, simplex)
library(rgl)
points3d(sims)
```

---

runif\_in\_tetrahedron *Uniform sampling in a tetrahedron*

---

**Description**

Uniform sampling in a tetrahedron (in dimension 3).

**Usage**

```
runif_in_tetrahedron(n, v1, v2, v3, v4)
```

**Arguments**

n                    number of simulations  
v1, v2, v3, v4      vertices of the tetrahedron

**Value**

The simulations in a n times 3 matrix.

**See Also**

[runif\\_in\\_simplex](#) for sampling in a simplex in arbitrary dimension.

**Examples**

```
library(rgl)
tetrahedron <- tetrahedron3d()
shade3d(tetrahedron, color = "red", alpha = 0.3)
vs <- tetrahedron$vb[1L:3L, ]
sims <- runif_in_tetrahedron(100, vs[, 1], vs[, 2], vs[, 3], vs[, 4])
points3d(sims)
```

---

runif\_on\_spherePatch *Uniform sampling on a spherical patch*

---

**Description**

Uniform sampling on a spherical patch (in dimension 3).

**Usage**

```
runif_on_spherePatch(n, r = 1, phi1, phi2, theta1, theta2)
```



**Arguments**

n                    number of simulations  
r                    radius  
phi1, phi2        numbers defining the latitudinal angle range  
theta1, theta2    numbers defining the longitudinal angle range

**Details**

A sphere patch is the part of the sphere whose polar angles theta and phi satisfy  $0 \leq \theta \leq \theta_1 \leq \theta_2 \leq 2\pi$  and  $0 \leq \phi \leq \phi_1 \leq \phi_2 \leq \pi$ .

**Value**

The simulations in a n times 3 matrix.

**See Also**

[runif\\_on\\_stri](#) for sampling on a spherical triangle.

**Examples**

```
# sampling on the first orthant:
sims <-
  runif_on_spherePatch(100, phi1 = 0, phi2 = pi/2, theta1 = 0, theta2 = pi/2)
## Not run:
library(rgl)
spheres3d(0, 0, 0, color = "red", alpha = 0.5)
points3d(sims)
## End(Not run)
```

---

runif\_on\_sphericalCap *Uniform sampling on a spherical cap*

---

**Description**

Uniform sampling on a spherical cap (in dimension 3).

**Usage**

```
runif_on_sphericalCap(n, r = 1, h)
```

**Arguments**

n                    number of simulations  
r                    radius of the sphere  
h                    height of the cap

**Value**

The simulations in a n times 3 matrix.

**Examples**

```
sims <- runif_on_sphericalCap(500, r = 2, h = 1)
## Not run:
library(rgl)
spheres3d(0, 0, 0, radius = 2, color = "red", alpha = 0.5)
points3d(sims)
## End(Not run)
```

---

runif\_on\_stri

*Uniform sampling on a spherical triangle*

---

**Description**

Uniform sampling on a spherical triangle (in dimension 3).

**Usage**

```
runif_on_stri(n, r = 1, v1, v2, v3)
```

**Arguments**

n	number of simulations
r	radius
v1, v2, v3	vertices

**Value**

The simulations in a n times 3 matrix.

**Examples**

```
# sampling on the first orthant:
sims <- runif_on_stri(100, v1 = c(1, 0, 0), v2 = c(0, 1, 0), v3 = c(0, 0, 1))
## Not run:
library(rgl)
spheres3d(0, 0, 0, color = "red", alpha = 0.5)
points3d(sims)
## End(Not run)
```

---

runif_sphere	<i>Uniform sampling on/in sphere</i>
--------------	--------------------------------------

---

**Description**

Uniform sampling on a sphere or in a sphere, in arbitrary dimension.

**Usage**

```
runif_on_sphere(n, d, r = 1)
```

```
runif_in_sphere(n, d, r = 1)
```

**Arguments**

n	number of simulations
d	dimension of the space
r	radius of the sphere

**Value**

The simulations in a n times d matrix.

**Examples**

```
sims <- runif_on_sphere(20, d = 2)
plot(sims, xlim = c(-1, 1), ylim = c(-1, 1), asp = 1, pch = 19)
sims <- runif_in_sphere(100, d = 2)
plot(sims, xlim = c(-1, 1), ylim = c(-1, 1), asp = 1, pch = 19)
```

---

runif_torus	<i>Uniform sampling on/in torus</i>
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---

**Description**

Uniform sampling on or in a torus (dimension 3).

**Usage**

```
runif_on_torus(n, R, r)
```

```
runif_in_torus(n, R, r)
```

**Arguments**

n	number of simulations
R	major radius
r	minor radius

**Value**

The simulations in a n times 3 matrix.

**Examples**

```
R <- 3; r <- 2
sims_on <- runif_on_torus(50, R = R, r = r)
sims_in <- runif_in_torus(50, R = R, r = r)
library(misc3d)
fx <- function(u,v) (R+r*cos(u)) * cos(v)
fy <- function(u,v) (R+r*cos(u)) * sin(v)
fz <- function(u,v) r*sin(u)
parametric3d(
  fx, fy, fz, umin = 0, umax = 2*pi, vmin = 0, vmax = 2*pi, alpha = 0.3
)
library(rgl)
points3d(sims_on)
points3d(sims_in, color = "red")
```

---

runif\_triangle

*Uniform sampling on/in a triangle*


---

**Description**

Uniform sampling on or in a triangle (dimension 2).

**Usage**

```
runif_in_triangle(n, v1, v2, v3)
```

```
runif_on_triangle(n, v1, v2, v3)
```

**Arguments**

n	number of simulations
v1, v2, v3	vertices of the triangle

**Value**

The simulations in a n times 2 matrix.

**Examples**

```
sims <- runif_on_triangle(30, c(0,0), c(1,0), c(0,1))
plot(sims, xlim = c(0,1), ylim = c(0,1), pch = 19)
sims <- runif_in_triangle(100, c(0,0), c(1,0), c(0,1))
plot(sims, xlim = c(0,1), ylim = c(0,1), pch = 19)
```

---

runif\_unitSimplex      *Uniform sampling on/in a unit simplex*

---

**Description**

Uniform sampling on or in a unit simplex (arbitrary dimension).

**Usage**

```
runif_on_unitSimplex(n, d)
runif_in_unitSimplex(n, d)
```

**Arguments**

n	number of simulations
d	dimension of the space

**Value**

The simulations in a n times d matrix.

**See Also**

[runif\\_in\\_tetrahedron](#) for sampling in an arbitrary tetrahedron in dimension 3; [runif\\_in\\_simplex](#) for sampling in an arbitrary simplex.

**Examples**

```
library(rgl)
sims <- runif_on_unitSimplex(300, d = 3)
points3d(sims)
```

surface\_sphere      *Sphere surface*

---

**Description**

Surface of a sphere (arbitrary dimension).

**Usage**

```
surface_sphere(d, r = 1)
```

**Arguments**

d                    dimension of the space  
r                    radius of the sphere

**Value**

The surface of the sphere of radius *r* in the *d*-dimensional space.

**Examples**

```
r <- 2  
surface_sphere(3, r)  
4*pi*r^2  
# perimeter of the unit circle:  
surface_sphere(2)
```

---

surface\_spherePatch      *Sphere patch surface*

---

**Description**

Surface of a sphere patch.

**Usage**

```
surface_spherePatch(r, phi1, phi2, theta1, theta2)
```

**Arguments**

r                    radius  
phi1, phi2          numbers defining the latitudinal angle range  
theta1, theta2      numbers defining the longitudinal angle range

**Details**

A sphere patch is the part of the sphere whose polar angles  $\theta$  and  $\phi$  satisfy  $0 \leq \theta_1 \leq \theta_2 \leq 2\pi$  and  $0 \leq \phi_1 \leq \phi \leq \phi_2 \leq \pi$ .

**Value**

The surface of the sphere patch.

**See Also**

[surface\\_stri](#) for the surface of a spherical triangle.

**Examples**

```
# surface of the first orthant:  
surface_spherePatch(r=1, phi1=0, phi2=pi/2, theta1=0, theta2=pi/2)  
surface_stri(r=1, c(1,0,0), c(0,1,0), c(0,0,1))
```

---

surface\_sphericalCap    *Spherical cap surface*

---

**Description**

Surface of a spherical cap.

**Usage**

```
surface_sphericalCap(r, h)
```

**Arguments**

r	radius of the sphere
h	height of the cap

**Value**

The surface area of the spherical cap.

---

surface_stri	<i>Spherical triangle surface</i>
--------------	-----------------------------------

---

**Description**

Surface of a spherical triangle.

**Usage**

```
surface_stri(r, v1, v2, v3)
```

**Arguments**

r	radius
v1, v2, v3	vertices

**Value**

The surface of the spherical triangle of radius r with vertices v1, v2, v3.

**Examples**

```
# surface of the first orthant:  
surface_stri(r=1, c(1,0,0), c(0,1,0), c(0,0,1))
```

---

surface_torus	<i>Torus surface</i>
---------------	----------------------

---

**Description**

Surface of a torus.

**Usage**

```
surface_torus(R, r)
```

**Arguments**

R	major radius
r	minor radius

**Value**

The surface area of the torus.



---

surface_triangle	<i>Triangle surface</i>
------------------	-------------------------

---

**Description**

Surface of a triangle.

**Usage**

```
surface_triangle(v1, v2, v3)
```

**Arguments**

v1, v2, v3      vertices of the triangle

**Value**

The surface of the triangle with vertices v1, v2, v3.

**Examples**

```
surface_triangle(c(0,0), c(0,1), c(1,0))
```

---

volume_ellipsoid	<i>Ellipsoid volume</i>
------------------	-------------------------

---

**Description**

Volume of an ellipsoid (arbitrary dimension).

**Usage**

```
volume_ellipsoid(A, r)
```

**Arguments**

A                    symmetric positive-definite matrix defining the ellipsoid (see Details)  
r                    "radius" (see Details)

**Details**

The (boundary of the) ellipsoid is the set of vectors  $x$  satisfying  $t(x) \%*\% A \%*\% x == r^2$ .

**Value**

The volume of the ellipsoid.

**Examples**

```
# dimension 2 (area), with diagonal matrix A
A <- diag(c(2,3))
r <- 2
volume_ellipsoid(A, r)
pi * r^2 / sqrt(A[1,1]*A[2,2])
```

---

volume_pball	<i>p-ball volume</i>
--------------	----------------------

---

**Description**

Euclidean volume of a p-ball (arbitrary dimension).

**Usage**

```
volume_pball(d, p, r = 1)
```

**Arguments**

d	dimension
p	exponent in the p-norm, a positive number
r	radius of the ball

**Value**

The volume of the p-ball with radius r.

**Examples**

```
volume_pball(d=4, p=2, r=2)
volume_sphere(d=4, r=2)
```

---

volume_simplex	<i>Simplex volume</i>
----------------	-----------------------

---

**Description**

Volume of a simplex (arbitrary dimension).

**Usage**

```
volume_simplex(simplex)
```

**Arguments**

simplex            a (d+1) times d matrix giving the vertices of the simplex (rows)

**Value**

The volume of the simplex.

**Examples**

```
set.seed(666)
simplex <- matrix(rnorm(4*3), nrow=4, ncol=3)
volume_simplex(simplex)
volume_tetrahedron(simplex[1,], simplex[2,], simplex[3,], simplex[4,])
```

---

volume_sphere	<i>Sphere volume</i>
---------------	----------------------

---

**Description**

Volume of a sphere (arbitrary dimension).

**Usage**

```
volume_sphere(d, r = 1)
```

**Arguments**

d                    dimension of the space  
r                    radius of the sphere

**Value**

The volume of the sphere with radius r in the d-dimensional space.

**Examples**

```
r <- 2
volume_sphere(3, r)
4/3*pi*r^3
```

---

volume\_sphericalCap    *Spherical cap volume*

---

**Description**

Volume of a spherical cap.

**Usage**

```
volume_sphericalCap(r, h)
```

**Arguments**

r	radius of the sphere
h	height of the cap

**Value**

The volume of the spherical cap.

---

volume\_tetrahedron    *Tetrahedron volume*

---

**Description**

Volume of a tetrahedron (dimension 3).

**Usage**

```
volume_tetrahedron(v1, v2, v3, v4)
```

**Arguments**

v1, v2, v3, v4	vertices of the tetrahedron
----------------	-----------------------------

**Value**

The volume of the tetrahedron.

**See Also**

[volume\\_simplex](#) for the volume of a simplex in arbitrary dimension.

**Examples**

```
v1 <- c(0,0,0); v2 <- c(1,0,0); v3 <- c(0,1,0); v4 <- c(0,0,1)
volume_tetrahedron(v1, v2, v3, v4)
volume_unitSimplex(3)
```

---

volume_torus	<i>Torus volume</i>
--------------	---------------------

---

**Description**

Volume of a torus.

**Usage**

```
volume_torus(R, r)
```

**Arguments**

R	major radius
r	minor radius

**Value**

The volume of the torus.

---

volume_unitSimplex	<i>Unit simplex volume</i>
--------------------	----------------------------

---

**Description**

Volume of the unit simplex (arbitrary dimension).

**Usage**

```
volume_unitSimplex(d)
```

**Arguments**

d	dimension of the space
---	------------------------

**Value**

The volume of the unit simplex in the space of dimension d.

**See Also**

[volume\\_simplex](#) for the volume of an arbitrary simplex.

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