Package ‘Tlasso’

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Description

Generate a matrix of bias-corrected sample covariance of residuals (excludes diagonal) described in Lyu et al. (2019).

Usage

biascor(rho, Omega.list, k = 1)

Arguments

rho: matrix of sample covariance of residuals (includes diagonal), e.g., output of covres.

Omega.list: list of precision matrices of tensor, i.e., Omega.list[[k]] is the precision matrix for the kth tensor mode, \( k \in \{1, \ldots, K\} \). For example, output of link{Tlasso.fit}.

k: index of interested mode, default is 1.

Details

This function computes bias-corrected sample covariance of residuals (excludes diagonal, diagonal is zero vector). Note that output matrix excludes diagonal while sample covariance of residuals includes diagonal, see Lyu et al. (2019) for details. Elements in Omega.list are true precision matrices or estimation of the true ones, the latter can be output of Tlasso.fit.

Value

A matrix whose (i,j) entry (excludes diagonal; diagonal is zero vector) is bias-corrected sample covariance of the ith and jth residuals in the kth mode. See Lyu et al. (2019) for details.

Author(s)

Xiang Lyu, Will Wei Sun, Zhaoran Wang, Han Liu, Jian Yang, Guang Cheng.

See Also

varcor, covres
### ChainOmega

**Description**

Generate precision matrix of triangle graph (chain like network) following the set-up in Fan et al. (2009).

**Usage**

```r
ChainOmega(p, sd = 1, norm.type = 2)
```

**Arguments**

- `p` dimension of generated precision matrix.
- `sd` seed for random number generation, default is 1.
- `norm.type` normalization methods of generated precision matrix, i.e., $\Omega_{11} = 1$ if `norm.type = 1` and $\|\Omega\|_F = 1$ if `norm.type = 2`. Default value is 2.

**Details**

This function first construct a covariance matrix $\Sigma$ that its $(i,j)$ entry is $\exp(-|h_i - h_j|/2)$ with $h_1 < h_2 < \ldots < h_p$. The difference $h_i - h_{i+1}$ is generated i.i.d. from $\text{Unif}(0,5,1)$. See Fan et al. (2009) for more details.

**Value**

A precision matrix generated from triangle graph.
Author(s)
Xiang Lyu, Will Wei Sun, Zhaoran Wang, Han Liu, Jian Yang, Guang Cheng.

See Also
NeighborOmega

Examples

```r
m.vec = c(5,5,5) # dimensionality of a tensor
n = 5   # sample size
Omega.true.list = list()
for ( k in 1:length(m.vec)){
  Omega.true.list[[k]] = ChainOmega(m.vec[k],sd=k*100,norm.type=2)
}
Omega.true.list # a list of length 3 contains precision matrices from triangle graph
```

covres

Sample Covariance Matrix of Residuals

Description
Generate sample covariance matrix of residuals (includes diagonal) described in Lyu et al. (2019).

Usage
covres(data, Omega.list, k = 1)

Arguments
data	 tensor object stored in a m1 * m2 * ... * mK * n array, where n is sample size and mk is dimension of the kth tensor mode.
Omega.list	 list of precision matrices of tensor, i.e., Omega.list[[k]] is precision matrix for the kth tensor mode, k ∈ {1,...,K}.
k	 index of interested mode, default is 1.

Details
This function computes sample covariance of residuals and is the basis for support recovery procedure in Lyu et al. (2019). Note that output matrix includes diagonal while bias corrected matrix (output of biascor) for inference is off-diagonal, see Lyu et al. (2019) for details. Elements in Omega.list are true precision matrices or estimation of the true ones, the latter can be output of Tlasso.fit.
**Value**

A matrix whose (i,j) entry (includes diagonal) is sample covariance of the ith and jth residuals in the kth mode. See Lyu et al. (2019) for details.

**Author(s)**

Xiang Lyu, Will Wei Sun, Zhaoran Wang, Han Liu, Jian Yang, Guang Cheng.

**See Also**

`varcor`, `biascor`

**Examples**

```r
m.vec = c(5,5,5)  # dimensionality of a tensor
n = 5       # sample size
k=1        # index of interested mode
lambda.thm = 20*c( sqrt(log(m.vec[1])/(n*prod(m.vec))),
                sqrt(log(m.vec[2])/(n*prod(m.vec))),
                sqrt(log(m.vec[3])/(n*prod(m.vec))))
DATA=Trnorm(n,m.vec,type='Var
      Chain'
)  # observations from tensor normal distribution
out.tlasso = Tlasso.fit(DATA,T=1,lambda.vec = lambda.thm)
# output is a list of estimation of precision matrices
rho=covres(DATA, out.tlasso, k = k)  # sample covariance of residuals, including diagonal
rho
```

---

**Description**

Compute estimation errors and TPR/TNR of optimization for sparse tensor graphical models

**Usage**

```r
est.analysis(Omega.hat.list, Omega.true.list, offdiag = TRUE)
```

**Arguments**

- `Omega.hat.list`: list of estimation of precision matrices of tensor, i.e., `Omega.hat.list[[k]]` is estimation of precision matrix for the kth tensor mode, \( k \in \{1, \ldots, K\} \). For example, output of `Tlasso.fit`.

- `Omega.true.list`: list of true precision matrices of tensor, i.e., `Omega.true.list[[k]]` is true precision matrix for the kth tensor mode, \( k \in \{1, \ldots, K\} \).
offdiag logical; indicate if excludes diagonal when computing performance measures. If offdiag = TRUE, diagonal in each matrix is ignored when comparing two matrices. Default is TRUE.

Details

This function computes performance measures of optimization for sparse tensor graphical models. Errors are measured in Frobenius norm and Max norm. Model selection measures are TPR and TNR. All these measures are computed in each mode, average across all modes, and kronecker product of precision matrices.

Value

A list, named Out, of following performance measures:

- Out$error.kro error in Frobenius norm of kronecker product
- Out$tpr.kro TPR of kronecker product
- Out$tnr.kro TNR of kronecker product
- Out$error.f vector; error in Frobenius norm of each mode
- Out$error.max vector; error in Max norm of each mode
- Out$tpr vector; TPR of each mode
- Out$tnr vector; TNR of each mode

Author(s)

Xiang Lyu, Will Wei Sun, Zhaoran Wang, Han Liu, Jian Yang, Guang Cheng.

See Also

Tlasso.fit, NeighborOmega, ChainOmega

Examples

m.vec = c(5,5,5) # dimensionality of a tensor
n = 5 # sample size
k=1 # index of interested mode
Omega.true.list = list()
Omega.true.list[[1]] = ChainOmega(m.vec[1], sd = 1)
Omega.true.list[[2]] = ChainOmega(m.vec[2], sd = 2)
Omega.true.list[[3]] = ChainOmega(m.vec[3], sd = 3)
lambda.thm = 20*sqrt(log(m.vec[1]/(n*prod(m.vec))),
                 sqrt(log(m.vec[2]/(n*prod(m.vec))),
                 sqrt(log(m.vec[3]/(n*prod(m.vec))))))
DATA=Tnform(n,m.vec,type='Chain')
# observations from tensor normal distribution
out.lasso = Tlasso.fit(DATA, T=1, lambda.vec = lambda.thm)
# output is a list of estimation of precision matrices
est.analysis(out.lasso, Omega.true.list, offdiag=TRUE)
# generate a list of performance measures

description

Draw an undirected graph based on precision matrix to present connection among variables.

Usage

graph.pattern(
  mat,
  main = NULL,
  edge.color = "gray50",
  vertex.color = "red",
  vertex.size = 3,
  vertex.label = NA,
  thres = 1e-05
)

Arguments

mat precision matrix that encodes information of graph structure.
main main title of graph. Default is NULL.
edge.color color of edge. Default is "gray50".
vertex.color color of vertex. Default is "red".
vertex.size size of vertex. Default is 3.
vertex.label label of vertex. Default is NA.
thres thresholding level of substituting entry with zero, set entry to zero if its absolute value equals or is less than thres. If thres is negative or zero, no entry will be substituted with zero.

Details

This function generates an undirected graph based on precision matrix. If an entry is zero, then no edge connects corresponding pair of nodes.

Value

A plot of undirected graph.
Author(s)
Xiang Lyu, Will Wei Sun, Zhaoran Wang, Han Liu, Jian Yang, Guang Cheng.

See Also
infer.analysis, est.analysis

Examples

```r
graph.pattern(ChainOmega(5, sd = 13))
# a triangle graph
```

---

infer.analysis  

**Inference Performance Measures**

Description
False positive, false negative, discoveries, and non-discoveries of inference for sparse tensor graphical models.

Usage

```r
infer.analysis(mat.list, critical, Omega.true.list, offdiag = TRUE)
```

Arguments

- **mat.list**
  list of matrices. (i,j) entry in its kth element is test statistic value for (i,j) entry of kth true precision matrix.

- **critical**
  critical level of rejecting null hypothesis. If critical is not positive, all null hypothesis will not be rejected.

- **Omega.true.list**
  list of true precision matrices of tensor, i.e., Omega.true.list[[k]] is true precision matrix for the kth tensor mode, $k \in \{1, \ldots, K\}$.

- **offdiag**
  logical; indicate if excludes diagonal when computing performance measures. If offdiag = TRUE, diagonal in each matrix is ignored when comparing two matrices. Default is TRUE.

Details
This function computes performance measures of inference for sparse tensor graphical models. False positive, false negative, discovery (number of rejected null hypothesis), non-discovery (number of non-rejected null hypothesis), and total non-zero entries of each true precision matrix is listed in output.
**Value**

A list, named `Out`, of following performance measures:
Out$fp  vector; number of false positive of each mode
Out$fn  vector; number of false negative of each mode
Out$d  vector; number of all discovery of each mode
Out$nd  vector; number of all non-discovery of each mode
Out$t  vector; number of all true non-zero entries in true precision matrix of each mode

Author(s)
Xiang Lyu, Will Wei Sun, Zhaoran Wang, Han Liu, Jian Yang, Guang Cheng.

See Also
Tlasso.fit, est.analysis, ChainOmega

Examples

m.vec = c(5,5,5)  # dimensionality of a tensor
n = 5  # sample size
Omega.true.list = list()
Omega.true.list[[1]] = ChainOmega(m.vec[1], sd = 1)
Omega.true.list[[2]] = ChainOmega(m.vec[2], sd = 2)
Omega.true.list[[3]] = ChainOmega(m.vec[3], sd = 3)
lambda.thm = 20*c(sqrt(log(m.vec[1])/(n*prod(m.vec))),
                   sqrt(log(m.vec[2])/(n*prod(m.vec))),
                   sqrt(log(m.vec[3])/(n*prod(m.vec))))
DATA=Trnorm(n,m.vec,type='Var')
# observation from tensor normal distribution
out.tlasso = Tlasso.fit(DATA,T=1,lambda.vec = lambda.thm)
# output is a list of estimation of precision matrices
mat.list=list()
for ( k in 1:3) {
  rho=covres(DATA, out.tlasso, k = k)
  # sample covariance of residuals, including diagonal
  varpi2=varcor(DATA, out.tlasso, k = k)
  # variance correction term for kth mode's sample covariance of residuals
  bias_rho=biascor(rho, out.tlasso, k=k)
  # bias corrected
  tautest=matrix(0,m.vec[k],m.vec[k])
  for( i in 1:(m.vec[k]-1)) {
    for ( j in (i+1):m.vec[k]){
      tautest[j,i]=tautest[i,j]=sqrt((n-1)*prod(m.vec[-k]))*
                      bias_rho[i,j]/sqrt(varpi2*rho[i,i]*rho[j,j])
    }
  }
  # list of matrices of test statistic values (off-diagonal). See Sun et al. 2016
  mat.list[[k]]=tautest
}

infer.analysis(mat.list, qnorm(0.975), Omega.true.list, offdiag=TRUE)
# inference measures (off-diagonal)

## NeighborOmega

**Description**
Generate precision matrix of nearest-neighbor network following the set-up in Li and Gui (2006) and Lee and Liu (2006).

**Usage**

NeighborOmega(p, sd = 1, knn = 4, norm.type = 2)

**Arguments**
- `p`: dimension of generated precision matrix.
- `sd`: seed for random number generation. Default is 1.
- `knn`: sparsity of precision matrix, i.e., matrix is generated from a knn nearest-neighbor graph. knn should be less than `p`. Default is 4.
- `norm.type`: normalization methods of generated precision matrix, i.e., $\Omega_{11} = 1$ if `norm.type` = 1 and $\|\Omega\|_F = 1$ if `norm.type` = 2. Default value is 2.

**Details**
For a knn nearest-neighbor graph, this function first randomly picks `p` points from a unit square and computes all pairwise distances among the points. Then it searches for the knn nearest-neighbors of each point and a pair of symmetric entries in the precision matrix that has a random chosen value from $[-1, -0.5] \cup [0.5, 1]$. Finally, to ensure positive definite property, it normalizes the matrix as $\Omega < -\Omega + (\lambda(\Omega) + 0.2)1_p$, where $\lambda(\cdot)$ refers to the smallest eigenvalue.

**Value**
A precision matrix generated from the knn nearest-neighbor graph.

**Author(s)**
Xiang Lyu, Will Wei Sun, Zhaoran Wang, Han Liu, Jian Yang, Guang Cheng.

**See Also**
ChainOmega
Examples

```r
m.vec = c(5,5,5)  # dimensionality of a tensor
n = 5   # sample size
knn=4  # sparsity

Omega.true.list = list()
for ( k in 1:length(m.vec)){
  Omega.true.list[[k]] = NeighborOmega(m.vec[k],knn=4, sd=k*100,norm.type=2)
}
Omega.true.list # a list of length 3 contains precision matrices from 4-nearest neighbor graph
```

---

**signal**  

*Regression Parameter of Conditional Linear Model*

**Description**

Compute regression parameter of conditional linear model of separable tensor normal distribution described in Lyu et al. (2019).

**Usage**

```r
signal(Omega.list, i = 1, k = 1)
```

**Arguments**

- **Omega.list** list of precision matrices of tensor, i.e., Omega.list[[k]] is the kth precision matrix. Omega.list can be either true precision matrices or output of Tlasso.fit. for the kth tensor mode, \( k \in \{1, \ldots, K\} \).

- **i** index of interested regression parameter, default is 1. See details in Lyu et al. (2019).

- **k** index of interested mode, default is 1.

**Details**

This function computes regression parameter and is fundamental for sample covariance of residuals and bias correction. See details in Lyu et al. (2019).

**Value**

A vector of regression parameter.

**Author(s)**

Xiang Lyu, Will Wei Sun, Zhaoran Wang, Han Liu, Jian Yang, Guang Cheng.
Tlasso

## See Also

covres, biascor

## Examples

```r
m.vec = c(5,5,5) # dimensionality of a tensor  
n = 5  # sample size  
k=1 # index of interested mode  
lambda.thm = 20*c( sqrt(log(m.vec[1])/(n*prod(m.vec))),  
                 sqrt(log(m.vec[2])/(n*prod(m.vec))),  
                 sqrt(log(m.vec[3])/(n*prod(m.vec))))  
DATA=Trnorm(n,m.vec,type='Var')  
# observations from tensor normal distribution  
out.tlasso = Tlasso.fit(DATA,T=1,lambda.vec = lambda.thm)  
# output is a list of estimation of precision matrices  
signal(out.tlasso, i=2 , k=k )  
# the regression parameter for conditional linear model of 2rd row in 1st mode
```

### Description

An optimal alternating optimization algorithm for estimation of precision matrices of sparse tensor graphical models, and an efficient inference procedure for support recovery of the precision matrices.

### Details

- **Package:** Tlasso
- **Type:** Package
- **Date:** 2016-09-17
- **License:** GPL (>= 2)

### Author(s)

Xiang Lyu, Will Wei Sun, Zhaoran Wang, Han Liu, Jian Yang, Guang Cheng.
Maintainer: Xiang Lyu <xianglyu@berkeley.edu>
Non-Convex Optimization for Sparse Tensor Graphical Models

Description

An alternating optimization algorithm for estimation of precision matrices of sparse tensor graphical models. See Lyu et al. (2019) for details.

Usage

```
Tlasso.fit(data, T = 1, lambda.vec = NULL, norm.type = 2, thres = 1e-05)
```

Arguments

data: tensor object stored in a m1 * m2 * ... * mK * n array, where n is sample size and mk is dimension of the kth tensor mode.

T: number of maximal iteration, default is 1. Each iteration involves update on all modes. If output change less than thres after certain iteration, in terms of summation on Frobenius norm, this function will be terminated (before Tth iteration).

lambda.vec: vector of tuning parameters (λ1,...,λK). Default is NULL, s.t. it is tuned via HUGE package directly.

norm.type: normalization method of precision matrix, i.e., Ω11 = 1 if norm.type = 1 and ∥Ω∥F = 1 if norm.type = 2. Default value is 2.

thres: thresholding value that terminates algorithm before Tth iteration if output change less than thres after certain iteration, in terms of summation over Frobenius norm. If thres is negative or zero, this algorithm will iterate T times.

Details

This function conducts an alternating optimization algorithm to sparse tensor graphical model. The output is optimal consistent even when T=1, see Lyu et al. (2019) for details. There are two termination criteria, T and thres. Algorithm will be terminated if output in certain iteration change less than thres. Otherwise, T iterations will be fully operated.
**Value**

A length-\(K\) list of estimation of precision matrices.

**Author(s)**

Xiang Lyu, Will Wei Sun, Zhaoran Wang, Han Liu, Jian Yang, Guang Cheng.

**See Also**

`varcor`, `biascor`, `huge`

**Examples**

```r
m.vec = c(5,5,5)  # dimensionality of a tensor
n = 5             # sample size
lambda.thm = 20*c(  
  sqrt(log(m.vec[1])/(n*prod(m.vec))),
  sqrt(log(m.vec[2])/(n*prod(m.vec))),
  sqrt(log(m.vec[3])/(n*prod(m.vec))))
DATA=Trnorm(n,m.vec,type='Var
Chain')
# obersavations from tensor normal distribution
out.tlasso = Tlasso.fit(DATA,T=10,lambda.vec = lambda.thm,thres=10)
# terminate by thres
out.tlasso = Tlasso.fit(DATA,T=3,lambda.vec = lambda.thm,thres=0)
# thres=0, iterate 10 times
```
Arguments

- **n**: number of generated observations.
- **m.vec**: vector of tensor mode dimensions, e.g., `m.vec=c(m1, m2, m3)` for a 3-mode tensor normal distribution.
- **mu**: array of mean for tensor normal distribution with dimension `m.vec`. Default is zero mean.
- **Sigma.list**: list of covariance matrices in mode sequence. Default is `NULL`.
- **type**: type of precision matrix, default is 'Chain'. Optional values are 'Chain' for triangle graph and 'Neighbor' for nearest-neighbor graph. Useless if `Sigma.list` is not `NULL`.
- **sd**: seed of random number generation, default is 1.
- **knn**: sparsity of precision matrix, i.e., matrix is generated from a knn nearest-neighbor graph. Default is 4. Useless if `type`='Chain' or `Sigma.list` is not `NULL`.
- **norm.type**: normalization method of precision matrix, i.e., $\Omega_{11} = 1$ if `norm.type` = 1 and $\|\Omega\|_F = 1$ if `norm.type` = 2. Default value is 2.

Details

This function generates observations from separable tensor normal distribution and returns a $m_1 \times \ldots \times m_K \times n$ array. If `Sigma.list` is not given, default distribution is from either triangle graph or nearest-neighbor graph (depends on `type`).

Value

An array with dimension $m_1 \times \ldots \times m_K \times n$.

Author(s)

Xiang Lyu, Will Wei Sun, Zhaoran Wang, Han Liu, Jian Yang, Guang Cheng.

See Also

- `ChainOmega`
- `NeighborOmega`

Examples

```r
m.vec = c(5,5,5)  # dimensionality of a tensor
n = 5             # sample size
DATA=Trnorm(n,m.vec,type='Chain')
# a 5*5*5*10 array of observation from 5*5*5 separable tensor
# normal distribution with mean zero and
# precision matrices from triangle graph
```
**Description**

Generate variance correction term of sample covariance of residuals described in Lyu et al. (2019).

**Usage**

```r
varcor(data, Omega.list, k = 1)
```

**Arguments**

- `data` tensor object stored in a `m1 * m2 * ... * mK * n` array, where `n` is sample size and `mk` is dimension of the kth tensor mode.
- `Omega.list` list of precision matrices of tensor, i.e., `Omega.list[[k]]` is precision matrix for the kth tensor mode, `k ∈ {1,...,K}`. Elements in `Omega.list` are true precision matrices or estimation of the true ones, the latter can be output of `Tlasso.fit`.
- `k` index of interested mode, default is 1.

**Details**

This function computes variance correction term of sample covariance of residuals and is utilized to normalize test statistic into standord normal, see Lyu et al. (2019).

**Value**

A scalar of variance correction for the kth mode.

**Author(s)**

Xiang Lyu, Will Wei Sun, Zhaoran Wang, Han Liu, Jian Yang, Guang Cheng.

**See Also**

- `varcor`, `biascor`, `covres`

**Examples**

```r
m.vec = c(5,5,5)  # dimensionality of a tensor
n = 5  # sample size
k=1  # index of interested mode
lambda.thm = 20*c( sqrt(log(m.vec[1]/(n*prod(m.vec)))),
                   sqrt(log(m.vec[2]/(n*prod(m.vec)))),
                   sqrt(log(m.vec[3]/(n*prod(m.vec)))))
DATA=Trnorm(n,m.vec,type='Chain')
```
# observations from tensor normal distribution
out.tlasso = Tlasso.fit(DATA, T=1, lambda.vec = lambda.thm)
# output is a list of estimation of precision matrices

rho = covres(DATA, out.tlasso, k = k)
# sample covariance of residuals, including diagonal
varpi2 = varcor(DATA, out.tlasso, k = k)
# variance correction term for kth mode's sample covariance of residuals
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