Package ‘KEPTED’

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Type    Package
Title   Kernel-Embedding-of-Probability Test for Elliptical Distribution
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BugReports https://github.com/tyy20/KEPTED/issues
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Description Provides an implementation of a kernel-embedding of probability test for elliptical distribution. This is an asymptotic test for elliptical distribution under general alternatives, and the location and shape parameters are assumed to be unknown. Some side-products are posted, including the transformation between rectangular and polar coordinates and two product-type kernel functions. See Tang and Li (2024) <doi:10.48550/arXiv.2306.10594> for details.
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EllKEPT

Description

This function gives a test on whether the data is elliptically distributed based on kernel embedding of probability. See Tang and Li (2024) for details. Gaussian kernels and product-type inverse quadratic kernels are considered.

Usage

EllKEPT(
  X,
  eps = 1e-06,
  kerU = "Gaussian",
  kerTheta = "Gaussian",
  gamma.U = 0,
  gamma.Theta = 0
)

Arguments

X
  A matrix with n rows and d columns.
eps
  The regularization constant added to the diagonal to avoid singularity. Default value is 1e-6.
kernU
  The type of kernel function on U. Currently supported options are "Gaussian" and "PIQ".
kernTheta
  The type of kernel function on Theta. Currently supported options are "Gaussian" and "PIQ".
gamma.U
  The tuning parameter gamma in the kernel function k_U(u1,u2). If gamma.U=0, the recommended procedure of selecting tuning parameter will be applied. Otherwise, the value given in gamma.U will be directly used as the tuning parameter. Default value is gamma.U=0. See "Details" for more information.
gamma.Theta
  The tuning parameter gamma in the kernel function k_Theta(theta1,theta2). If gamma.Theta=0, the recommended procedure of selecting tuning parameter will be applied. Otherwise, the value given in gamma.Theta will be directly used as the tuning parameter. Default value is gamma.Theta=0. See "Details" for more information.
**Details**

The Gaussian kernel is defined as \(k(z_1, z_2) = \exp(-\gamma \|z_1 - z_2\|^2)\), and the Product-type Inverse-Quadratic (PIQ) kernel is defined as \(k(z_1, z_2) = \prod_j (1/(1 + \gamma (z_{1,j} - z_{2,j})^2))\). The recommended procedure of selecting tuning parameter is given as in the simulation section of Tang and Li (2023+), where we set \(1/\sqrt{\gamma} = (n(n-1)/2)^{-1} \sum_{1 \leq i < j \leq n} \|Z_i - Z_j\|\).

**Value**

A list of the following:

- **stat** The value of the test statistic.
- **pval** The p-value of the test.
- **lambda** The \(n\) eigenvalues in the approximated asymptotic distribution.
- **gamma.U** The tuning parameter \(\gamma_U\) used in the test. Same as the input if its input is nonzero.
- **gamma.Theta** The tuning parameter \(\gamma_{\Theta}\) used in the test. Same as the input if its input is nonzero.

**Note**

In the arguments, \(\epsilon\) refers to a regularization constant added to the diagonal. When the dimension is high, we recommend increasing \(\epsilon\) to avoid singularity.

**References**


**Examples**

```r
set.seed(313)
n=50
d=3

## Null Hypothesis
X=matrix(rnorm(n*d),nrow=n,ncol=d)
EllKEPT(X)

## Alternative Hypothesis
X=matrix(rchisq(n*d,2)-2,nrow=n,ncol=d)
EllKEPT(X)
```
**Description**

Provides an implementation of a kernel-embedding of probability test for elliptical distribution. This is an asymptotic test for elliptical distribution under general alternatives, and the location and shape parameters are assumed to be unknown. Some side-products are posted, including the transformation between rectangular and polar coordinates and two product-type kernel functions.

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**References**


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**kerGauss**

*Gaussian kernel computation*

**Description**

Computing the values of Gaussian kernel functions.

**Usage**

kerGauss(gamma, z1, z2)

**Arguments**

- **gamma**: A number, the bandwidth parameter in the Gaussian kernel.
- **z1** : A vector, the first input of the Gaussian kernel.
- **z2** : A vector, the second input of the Gaussian kernel.

**Details**

The Gaussian kernel is defined as \( k(z_1, z_2) = \exp(-\gamma \|z_1 - z_2\|^2) \).
Value

A number, the value of the Gaussian kernel function.

Examples

gamma=0.02
z1=c(3,1,3)
z2=c(8,1,9)
kerGauss(gamma,z1,z2)

kerPIQ

Product-type Inverse-Quadratic (PIQ) kernel computation

Description

Computing the values of Product-type Inverse-Quadratic (PIQ) kernel functions.

Usage

kerPIQ(gamma, z1, z2)

Arguments

gamma A number, the bandwidth parameter in the PIQ kernel.
z1 A vector, the first input of the PIQ kernel.
z2 A vector, the second input of the PIQ kernel.

Details

The Product-type Inverse-Quadratic (PIQ) kernel is defined as \( k(z1,z2) = \prod_j \frac{1}{1 + \gamma (z1_j - z2_j)^2} \).

Value

A number, the value of the PIQ kernel function.

Examples

gamma=0.02
z1=c(3,1,3)
z2=c(8,1,9)
kerPIQ(gamma,z1,z2)
Polar2Rec  

**Polar to rectangular coordinates**

**Description**

Given a polar coordinate representation \((R, \Theta)\) of a \(d\)-dimensional vector \(X\), where \(R\) is the length of \(X\) and the \((d-1)\)-dimensional vector \(\Theta\) contains the \(d-1\) angles of \(X\), this function compute \(X\) in its rectangular coordinate representation.

**Usage**

\[
Polar2Rec(R, \Theta)
\]

**Arguments**

- **R**
  - The length of \(X\).
- **Theta**
  - A vector of length \(d-1\), containing the angles of \(X\).

**Details**

The formula corresponds to \(v=\rho(\theta)\) as in Lemma 1 of Tang and Li (2024). See also Anderson (2003). Note that when \(d=2\), \(V\) will be \((\sin(\Theta), \cos(\Theta))\).

**Value**

A list of the following:

- **X**
  - A vector in rectangular coordinate.
- **V**
  - The directional vector of \(X\). Note that \(V\) is always on the unit sphere.

**References**


**Examples**

\[
R=2
Theta=c(pi/6, pi/3)
Polar2Rec(R, Theta)
\]
PolarDerivative

Derivative of the polar coordinate transformation

Description

This function computes the Jacobian matrix of the polar transformation \( \theta = g(v) \), i.e., the transformation from the rectangular coordinate representation of the directional vector to its angular representation.

Usage

PolarDerivative(v)

Arguments

- \( v \) A \( d \)-dimensional directional vector of length 1.

Details

See Lemma 3 of Tang and Li (2024).

Value

The Jacobian matrix of the polar transformation \( \theta = g(v) \), with \( d-1 \) rows and \( d \) columns.

References


Examples

```r
X=c(3,1,3)
V=X/sqrt(sum(X^2))
PolarDerivative(V)
```
Rec2Polar

Rectangular to polar coordinates

**Description**

Given a d-dimensional vector $X$ in rectangular coordinate, this function compute its polar coordinate $(R, \Theta)$, where $R$ is the length of $X$ and the $(d-1)$-dimensional vector $\Theta$ contains the $d-1$ angles of $X$.

**Usage**

Rec2Polar(X)

**Arguments**

- **X**  
  A vector in rectangular coordinate. Suppose the dimension of $X$ is $d$.

**Details**

The formula corresponds to $\theta=g(v)$ as in Lemma 1 of Tang and Li (2024). See also Anderson (2003). Note that when $d=2$, $V$ will be $(\sin(\Theta), \cos(\Theta))$.

**Value**

A list of the following:

- **R**  
  The length of $X$.

- **Theta**  
  A vector of length $d-1$, containing the angles of $X$.

**References**


**Examples**

X=c(3,1,3)  
Rec2Polar(X)
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