Package ‘HypergeoMat’

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Type Package
Title Hypergeometric Function of a Matrix Argument
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Description Evaluates the hypergeometric functions of a matrix argument, which appear in random matrix theory. This is an implementation of Koev & Edelman's algorithm (2006) <doi:10.1090/S0025-5718-06-01824-2>.
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BesselA

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Description

Evaluates the type one Bessel function of Herz.

Usage

BesselA(m, x, nu)

Arguments

- \( m \) truncation weight of the summation, a positive integer
- \( x \) either a real or complex square matrix, or a numeric or complex vector, the eigenvalues of the matrix
- \( \nu \) the order parameter, real or complex number with \( \text{Re}(\nu) \geq -1 \)

Value

A real or complex number.

Note

This function is usually defined for a symmetric real matrix or a Hermitian complex matrix.

References

Examples

  # for a scalar x, the relation with the Bessel J-function:
  t <- 2
  nu <- 3
  besselJ(t, nu)
  BesselA(m=15, t^2/4, nu) * (t/2)^nu
  # it also holds for a complex variable:
  t <- 1 + 2i
  Bessel::BesselJ(t, nu)
  BesselA(m=15, t^2/4, nu) * (t/2)^nu

hypergeomPFQ  Hypergeometric function of a matrix argument

Description

Evaluates a truncated hypergeometric function of a matrix argument.

Usage

hypergeomPFQ(m, a, b, x, alpha = 2)

Arguments

  m      truncation weight of the summation, a positive integer
  a      the "upper" parameters, a numeric or complex vector, possibly empty (or NULL)
  b      the "lower" parameters, a numeric or complex vector, possibly empty (or NULL)
  x      either a real or complex square matrix, or a numeric or complex vector, the
eigenvalues of the matrix
  alpha  the alpha parameter, a positive number

Details

This is an implementation of Koev & Edelman’s algorithm (see the reference). This algorithm is
split into two parts: the case of a scalar matrix (multiple of an identity matrix) and the general case.
The case of a scalar matrix is much faster (try e.g. x = c(1,1,1) vs x = c(1,1,0.999)).

Value

A real or a complex number.

Note

The hypergeometric function of a matrix argument is usually defined for a symmetric real matrix or
a Hermitian complex matrix.
References


Examples

```r
# a scalar x example, the Gauss hypergeometric function
hypergeomPFQ(m = 10, a = c(1,2), b = c(3), x = 0.2)
gsl::hyperg_2F1(1, 2, 3, 0.2)
# 0F0 is the exponential of the trace
X <- toeplitz(c(3,2,1))/10
hypergeomPFQ(m = 10, a = NULL, b = NULL, x = X)
exp(sum(diag(X)))
# 1F0 is det(I-X)^(-a)
X <- toeplitz(c(3,2,1))/100
hypergeomPFQ(m = 10, a = 3, b = NULL, x = X)
det(diag(3)-X)^(-3)
# Herz's relation for 1F1
hypergeomPFQ(m = 10, a = 2, b = 3, x = X)
exp(sum(diag(X))) * hypergeomPFQ(m = 10, a = 3-2, b = 3, x = -X)
# Herz's relation for 2F1
hypergeomPFQ(10, a = c(1,2), b = 3, x = X)
det(diag(3)-X)^(-2) * hypergeomPFQ(10, a = c(3-1,2), b = 3, -X %*% solve(diag(3)-X))
```

---

**IncBeta**  
*Incomplete Beta function of a matrix argument*

Description

Evaluates the incomplete Beta function of a matrix argument.

Usage

```r
IncBeta(m, a, b, x)
```

Arguments

- **m**: truncation weight of the summation, a positive integer
- **a, b**: real or complex parameters with Re(a)>(p-1)/2, Re(b)>>(p-1)/2, where p is the dimension (the order of the matrix)
- **x**: either a real positive symmetric matrix or a complex positive Hermitian matrix "smaller" than the identity matrix (i.e. I-x is positive), or a numeric or complex vector, the eigenvalues of the matrix

Value

A real or a complex number.
Note

The eigenvalues of a real symmetric matrix or a complex Hermitian matrix are always real numbers, and moreover they are positive under the constraints on \( x \). However we allow to input a numeric or complex vector \( x \) because the definition of the function makes sense for such a \( x \).

References


Examples

```r
# for a scalar x, this is the incomplete Beta function:
a <- 2; b <- 3
x <- 0.75
IncBeta(m = 15, a, b, x)
gsl::beta_inc(a, b, x)
pbeta(x, a, b)
```

Description

Evaluates the incomplete Gamma function of a matrix argument.

Usage

```r
IncGamma(m, a, x)
```

Arguments

- \( m \) : truncation weight of the summation, a positive integer
- \( a \) : real or complex parameter with \( \text{Re}(a) > (p-1)/2 \), where \( p \) is the dimension (the order of the matrix)
- \( x \) : either a real or complex square matrix, or a numeric or complex vector, the eigenvalues of the matrix

Value

A real or complex number.

Note

This function is usually defined for a symmetric real matrix or a Hermitian complex matrix.

References

Examples

# for a scalar x, this is the incomplete Gamma function:
a <- 2
x <- 1.5
IncGamma(m = 15, a, x)
gsl::gamma_inc_P(a, x)
pgamma(x, shape = a, rate = 1)

Description

The multivariate Beta function (mvbeta) and its logarithm (lmvbeta).

Usage

lmvbeta(a, b, p)
mvbeta(a, b, p)

Arguments

a, b real or complex numbers with Re(a)>0, Re(b)>0
p a positive integer, the dimension

Value

A real or a complex number.

Examples

a <- 5; b <- 4; p <- 3
mvbeta(a, b, p)
mvgamma(a, p) * mvgamma(b, p) / mvgamma(a+b, p)
mvgamma

Multivariate Gamma function (of complex variable)

Description
The multivariate Gamma function (mvgamma) and its logarithm (lmvgamma).

Usage
lmvgamma(x, p)
mvgamma(x, p)

Arguments
x a real or a complex number; Re(x)>0 for lmvgamma and x must not be a negative integer for mvgamma
p a positive integer, the dimension

Value
A real or a complex number.

Examples
x <- 5
mvgamma(x, p = 2)
sqrt(pi)*gamma(x)*gamma(x-1/2)
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