

Package ‘FuzzyResampling’

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Type Package

Title Resampling Methods for Triangular and Trapezoidal Fuzzy Numbers

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Description

The classical (i.e. Efron's, see Efron and Tibshirani (1994, ISBN:978-0412042317) ``An Introduction to the Bootstrap") bootstrap is widely used for both the real (i.e. ``crisp") and fuzzy data. The main aim of the algorithms implemented in this package is to overcome a problem with repetition of a few distinct values and to create fuzzy numbers, which are ``similar" (but not the same) to values from the initial sample.

To do this, different characteristics of triangular/trapezoidal numbers are kept (like the value, the ambiguity, etc., see Grzegorzewski et al. <[doi:10.2991/eusflat-19.2019.68](https://doi.org/10.2991/eusflat-19.2019.68)>, Grzegorzewski et al. (2020) <[doi:10.2991/ijcis.d.201012.003](https://doi.org/10.2991/ijcis.d.201012.003)>, Grzegorzewski et al. (2020) <[doi:10.34768/amcs-2020-0022](https://doi.org/10.34768/amcs-2020-0022)>, Grzegorzewski and Romaniuk (2022) <[doi:10.1007/978-3-030-95929-6_3](https://doi.org/10.1007/978-3-030-95929-6_3)>, Romaniuk and Hryniewicz (2019) <[doi:10.1007/s00500-018-3251-5](https://doi.org/10.1007/s00500-018-3251-5)>).

Some additional procedures related to these resampling methods are also provided, like calculation of the Bertoluzza et al.'s distance (aka the mid/spread distance, see Bertoluzza et al. (1995) ``On a new class of distances between fuzzy numbers") and estimation of the p-value of the one-sample bootstrapped test for the mean (see Lubiano et al. (2016, <[doi:10.1016/j.ejor.2015.11.016](https://doi.org/10.1016/j.ejor.2015.11.016)>)).

Additionally, there are procedures which randomly generate trapezoidal fuzzy numbers using some well-known statistical distributions.

Imports stats

URL <https://github.com/mroman-ibs/FuzzyResampling>

BugReports <https://github.com/mroman-ibs/FuzzyResampling/issues>

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BertoluzzaDistance	<i>Calculate Bertoluzza's (mid/spread) distance for triangular and trapezoidal fuzzy numbers</i>
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Description

BertoluzzaDistance returns the Bertoluzza et al.'s (aka mid/spread) distance with the given weight theta between two triangular or trapezoidal fuzzy numbers.

Usage

```
BertoluzzaDistance(fuzzyNumber1, fuzzyNumber2, theta = 1/3, increases = FALSE)
```

Arguments

fuzzyNumber1	The first triangular or trapezoidal fuzzy number. More than one value can be given in the form of matrix.
fuzzyNumber2	The second triangular or trapezoidal fuzzy number. More than one value can be given in the form of matrix.
theta	The weighting parameter for the mid/spread distance.
increases	If TRUE is used, then the fuzzy numbers should be given in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers should be given in the form: left end of the support, left end of the core, right end of the core, right end of the support.

Details

The input fuzzy values should be triangular or trapezoidal fuzzy numbers, given as a single vector or a whole matrix. In each row, there should be a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

In this second case, the parameter `increases=TRUE` has to be used.

The procedure calculates the mid/spread distance between two fuzzy values based on the weight `theta`, usually we have `theta = 1/3` or `theta = 1`. The output is given as vector of distances.

Value

This function returns vector of doubles related to the calculated distances.

References

Bertoluzza, C., Corral, N., Salas, A. (1995) On a new class of distances between fuzzy numbers *Mathware and Soft Computing*, 2 (2), pp. 71-84

Examples

```
# prepare some fuzzy numbers (first type of the initial sample)
fuzzyValues <- matrix(c(0.25,0.5,1,1.25,0.75,1,1.5,2.2,-1,0,0,2),ncol = 4,byrow = TRUE)

# calculate the mid/spread distance between the first value
# (from the first row) and the second one (from the second row)
BertoluzzaDistance(fuzzyValues[1,],fuzzyValues[2,])

# calculate the mid/spread distance between the first value
# (from the first row) and all of the values (from the first to the third row)
BertoluzzaDistance(fuzzyValues[1,],fuzzyValues)
```

classicalBootstrap	<i>Classical bootstrap procedure for triangular and trapezoidal fuzzy numbers</i>
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Description

`classicalBootstrap` returns the bootstrapped (secondary) sample based on the initial sample and uses the Efron's (i.e. classical) resampling scheme (see (Efron, 1994)).

Usage

```
classicalBootstrap(initialSample, b = n, increases = FALSE)
```

Arguments

<code>initialSample</code>	Initial sample of triangular or trapezoidal fuzzy numbers.
<code>b</code>	The number of fuzzy values in the resampled (secondary) sample. If this parameter is not specified, the number of values in the initial sample is used. The parameter should be integer value more than 0.
<code>increases</code>	If TRUE is used, then the initial sample should consist of the fuzzy numbers in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers should be given in the form: left end of the support, left end of the core, right end of the core, right end of the support.

Details

The initial sample should consist of triangular or trapezoidal fuzzy numbers, given as a single vector or a whole matrix. In each row, there should be a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

In this second case, the parameter `increases=TRUE` has to be used.

The resampling procedure produces `b` fuzzy values, which are randomly chosen (with repetition) from the initial sample (without any alternations). If the parameter `b` is not specified, it is equal to the length of the initial sample. The output is given in the same form as the initial sample.

Value

This function returns matrix with `b` rows of double values. In each row, there is a single resampled fuzzy number. These fuzzy numbers have the same form as the values from the initial sample depending on the provided parameter `increases`.

References

Efron, B. (1994). An Introduction to the Bootstrap. CRC Press

See Also

[VAMethod](#) for the VA method, [EWmethod](#) for the EW method, [VAFmethod](#) for the VAF method, [VAAmethod](#) for the VAA method, [dmethod](#) for the d method, [wmethod](#) for the w method

Other resampling functions: [EWmethod\(\)](#), [VAAmethod\(\)](#), [VAFmethod\(\)](#), [VAMethod\(\)](#), [dmethod\(\)](#), [wmethod\(\)](#)

Examples

```
# prepare some fuzzy numbers (first type of the initial sample)

fuzzyValues <- matrix(c(0.25,0.5,1,1.25,0.75,1,1.5,2.2,-1,0,0,2),
  ncol = 4,byrow = TRUE)

# generate bootstrap sample

set.seed(12345)

classicalBootstrap(fuzzyValues)

classicalBootstrap(fuzzyValues,b=4)

# prepare some fuzzy numbers (second type of the initial sample)

fuzzyValuesInc <- matrix(c(0.25,0.5,1,0.25,0.25,1,1.5,0.7,1,0,0,2),
  ncol = 4,byrow = TRUE)

# generate bootstrap sample

classicalBootstrap(fuzzyValuesInc,increases = TRUE)

classicalBootstrap(fuzzyValuesInc,b=4,increases = TRUE)
```

dmethod

d method for resampling triangular and trapezoidal fuzzy numbers

Description

dmethod returns the secondary (bootstrapped) sample and uses the resampling scheme which does not change the left end of the cores and increments (i.e. length of the core, left and right increment of the support) of the fuzzy variables from the initial sample (the d method, see (Romaniuk and Hryniewicz, 2019)).

Usage

```
dmethod(initialSample, b = n, increases = FALSE)
```

Arguments

initialSample Initial sample of triangular or trapezoidal fuzzy numbers.

b The number of fuzzy values in the resampled (secondary) sample. If this parameter is not specified, the number of values in the initial sample is used. The parameter should be integer value more than 0.

increases If TRUE is used, then the initial sample should consist of the fuzzy numbers in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers should be given in the form: left end of the support, left end of the core, right end of the core, right end of the support.

Details

The initial sample should consist of triangular or trapezoidal fuzzy numbers, given as a single vector or a whole matrix. In each row, there should be a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

In this second case, the parameter `increases=TRUE` has to be used.

The resampling procedure produces `b` fuzzy values. During the first step, the four values are randomly chosen based on the whole initial sample: left end of the core, length of the core, left and right increment of the support (with equal probabilities for each fuzzy variable). Then the new fuzzy variable, which preserves these characteristics, is created. If the parameter `b` is not specified, it is equal to the length of the initial sample. The output is given in the same form as the initial sample.

Value

This function returns matrix with `b` rows of double values. In each row, there is a single resampled fuzzy number. These fuzzy numbers have the same form as the values from the initial sample depending on the provided parameter `increases`.

References

Romaniuk, M., Hryniewicz, O. (2019) Interval-based, nonparametric approach for resampling of fuzzy numbers *Soft Computing*, 23 (14), pp. 5883-5903

See Also

@seealso [classicalBootstrap](#), [EWmethod](#) for the EW method, [VAFmethod](#) for the VAF method, [VAAmethod](#) for the VAA method, [wmethod](#) for the w method

Other resampling functions: [EWmethod\(\)](#), [VAAmethod\(\)](#), [VAFmethod\(\)](#), [VAmethod\(\)](#), [classicalBootstrap\(\)](#), [wmethod\(\)](#)

Examples

```
# prepare some fuzzy numbers (first type of the initial sample)

fuzzyValues <- matrix(c(0.25,0.5,1,1.25,0.75,1,1.5,2.2,-1,0,0,2),
  ncol = 4,byrow = TRUE)

# generate the secondary sample using the d method
```

```

set.seed(12345)

dmethod(fuzzyValues)

dmethod(fuzzyValues,b=4)

# prepare some fuzzy numbers (second type of the initial sample)

fuzzyValuesInc <- matrix(c(0.25,0.5,1,0.25,0.25,1,1.5,0.7,1,0,0,2),
ncol = 4,byrow = TRUE)

# generate the secondary sample using the d method

dmethod(fuzzyValuesInc,increases = TRUE)

dmethod(fuzzyValuesInc,b=4,increases = TRUE)

```

EWmethod	<i>E(xpected value)W(idth) resampling method for triangular and trapezoidal fuzzy numbers</i>
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Description

EWmethod returns the secondary (bootstrapped) sample and uses the resampling scheme which does not change the expected values and widths of the fuzzy variables from the initial sample (the EW method, see (Grzegorzewski et al, 2020)).

Usage

```
EWmethod(initialSample, b = n, increases = FALSE)
```

Arguments

initialSample	Initial sample of triangular or trapezoidal fuzzy numbers.
b	The number of fuzzy values in the resampled (secondary) sample. If this parameter is not specified, the number of values in the initial sample is used. The parameter should be integer value more than 0.
increases	If TRUE is used, then the initial sample should consist of the fuzzy numbers in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers should be given in the form: left end of the support, left end of the core, right end of the core, right end of the support.

Details

The initial sample should consist of triangular or trapezoidal fuzzy numbers, given as a single vector or a whole matrix. In each row, there should be a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

In this second case, the parameter `increases=TRUE` has to be used.

The resampling procedure produces `b` fuzzy values. During the first step, the fuzzy value from the initial sample is randomly chosen (with repetition). Then the new fuzzy variable, which preserves the expected value and width of the old one, is randomly created. If the parameter `b` is not specified, it is equal to the length of the initial sample. The output is given in the same form as the initial sample.

Value

This function returns matrix with `b` rows of double values. In each row, there is a single resampled fuzzy number. These fuzzy numbers have the same form as the values from the initial sample depending on the provided parameter `increases`.

References

Grzegorzewski, P., Hryniewicz, O., Romaniuk, M. (2020) Flexible resampling for fuzzy data International Journal of Applied Mathematics and Computer Science, 30 (2), pp. 281-297

See Also

[classicalBootstrap](#), [VAMethod](#) for the VA method, [VAFMethod](#) for the VAF method, [VAAMethod](#) for the VAA method, [dmethod](#) for the d method, [wmethod](#) for the w method

Other resampling functions: [VAAMethod\(\)](#), [VAFMethod\(\)](#), [VAMethod\(\)](#), [classicalBootstrap\(\)](#), [dmethod\(\)](#), [wmethod\(\)](#)

Examples

```
# prepare some fuzzy numbers (first type of the initial sample)

fuzzyValues <- matrix(c(0.25,0.5,1,1.25,0.75,1,1.5,2.2,-1,0,0,2),
  ncol = 4,byrow = TRUE)

# generate the secondary sample using the EW method

set.seed(12345)

EWmethod(fuzzyValues)

EWmethod(fuzzyValues,b=4)

# prepare some fuzzy numbers (second type of the initial sample)
```



```
fuzzyValuesInc <- matrix(c(0.25,0.5,1,0.25,0.25,1,1.5,0.7,1,0,0,2),
ncol = 4,byrow = TRUE)

# generate the secondary sample using the EW method

EWmethod(fuzzyValuesInc,increases = TRUE)

EWmethod(fuzzyValuesInc,b=4,increases = TRUE)
```

GeneratorNExpUU

Generate initial sample using the normal and uniform distributions.

Description

GeneratorNExpUU generates the random sample of trapezoidal fuzzy numbers using the normal, exponential and uniform distributions (for the "true" origin of each fuzzy number, and its increases, respectively).

Usage

```
GeneratorNExpUU(n, mu, sigma, lambda, b, c, increases = FALSE, ...)
```

Arguments

n	Number of fuzzy trapezoidal numbers in the created sample.
mu	The expected value of the normal distribution which is used to create the "true origin" of fuzzy trapezoidal number.
sigma	The standard deviation of the normal distribution which is used to create the "true origin" of fuzzy trapezoidal number.
lambda	The parameter of the exponential distributions which are used to create the left and right increases of the core.
b	The right end of the interval of the independent uniform distribution which is used to create the left increase of the support.
c	The right end of the interval of the independent uniform distribution which is used to create the right increase of the support.
increases	If TRUE is used, then the output fuzzy numbers are given in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers are given in the form: left end of the support, left end of the core, right end of the core, right end of the support.
...	Some additional parameters to pass to other functions.

Details

The procedure simulates the initial sample which consists of n trapezoidal fuzzy numbers. The "true origin" of each fuzzy number is independently drawn from the normal distribution $N(\mu, \sigma)$. Then increases of its core are independently generated from the exponential distributions $\text{Exp}(\lambda)$. The increases of its support are independently drawn from the uniform distributions $U[0, b]$ (the left-hand increase) and $U[0, c]$ (its right-hand counterpart) (see Grzegorzewski et al. (2020)).

The output is given as a matrix. In each row, there is a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

To obtain this second form, the parameter `increases=TRUE` has to be used.

Value

This function returns matrix which consists of n trapezoidal fuzzy numbers. Each fuzzy number (four values) is given as one row in this matrix.

References

Grzegorzewski, P., Hryniewicz, O., Romaniuk, M. (2020) Flexible resampling for fuzzy data based on the canonical representation *International Journal of Computational Intelligence Systems*, 13 (1), pp. 1650-1662

See Also

[GeneratorNU](#)

Other sampling functions: [GeneratorNU\(\)](#)

Examples

```
# seed PRNG
set.seed(1234)

# generate 10 trapezoidal fuzzy numbers
GeneratorNExpUU(10, 0, 1, 1, 1, 2)
```

GeneratorNU *Generate initial sample using the normal and uniform distributions.*

Description

GeneratorNU generates the random sample of trapezoidal fuzzy numbers using the normal and uniform distributions (for the "true" origin of each fuzzy number, and its increases, respectively).

Usage

GeneratorNU(*n*, *mu*, *sigma*, *a*, *b*, *increases* = FALSE, ...)

Arguments

<i>n</i>	Number of fuzzy trapezoidal numbers in the created sample.
<i>mu</i>	The expected value of the normal distribution which is used to create the "true origin" of fuzzy trapezoidal number.
<i>sigma</i>	The standard deviation of the normal distribution which is used to create the "true origin" of fuzzy trapezoidal number.
<i>a</i>	The right end of the interval of the independent uniform distributions which are used to create the left and right increases of the core.
<i>b</i>	The right end of the interval of the independent uniform distributions which are used to create the left and right increases of the support.
<i>increases</i>	If TRUE is used, then the output fuzzy numbers are given in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers are given in the form: left end of the support, left end of the core, right end of the core, right end of the support.
...	Some additional parameters to pass to other functions.

Details

The procedure simulates the initial sample which consists of *n* trapezoidal fuzzy numbers. The "true origin" of each fuzzy number is independently drawn from the normal distribution $N(\mu, \sigma)$. Then increases of its core and support are independently generated from the uniform distributions $U[0, a]$ and $U[0, b]$ (see Grzegorzewski et al. (2020)).

The output is given as a matrix. In each row, there is a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

To obtain this second form, the parameter *increases*=TRUE has to be used.

Value

This function returns matrix which consists of n trapezoidal fuzzy numbers. Each fuzzy number (four values) is given as one row in this matrix.

References

Grzegorzewski, P., Hryniewicz, O., Romaniuk, M. (2020) Flexible resampling for fuzzy data based on the canonical representation International Journal of Computational Intelligence Systems, 13 (1), pp. 1650-1662

See Also

[GeneratorNExpUU](#)

Other sampling functions: [GeneratorNExpUU\(\)](#)

Examples

```
# seed PRNG
set.seed(1234)

# generate 10 trapezoidal fuzzy numbers
GeneratorNU(10, 0,1,1,2)
```

OneSampleCTest

Calculate p-value of the one-sample test for the mean

Description

OneSampleCTest returns the p-value of the one-sample test for the mean using the resampling method.

Usage

```
OneSampleCTest(
  initialSample,
  mu_0,
  numberOfSamples = 100,
  theta = 1/3,
  resamplingMethod = classicalBootstrap,
  increases = FALSE
)
```

Arguments

<code>initialSample</code>	The initial sample which consists of triangular or trapezoidal fuzzy numbers. More than one value can be given in the form of matrix.
<code>mu_0</code>	Triangular or trapezoidal fuzzy number which is used for the null hypothesis of the C-test.
<code>numberOfSamples</code>	Number of the bootstrapped samples used to estimate the p-value.
<code>theta</code>	The weighting parameter for the mid/spread distance applied in the C-test.
<code>resamplingMethod</code>	Name of the resampling method, which is used to generate the bootstrapped samples. For the possible names check the values of <code>resamplingMethods</code> vector.
<code>increases</code>	If TRUE is used, then the fuzzy numbers should be given in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers should be given in the form: left end of the support, left end of the core, right end of the core, right end of the support.

Details

The input fuzzy values should be triangular or trapezoidal fuzzy numbers, given as a single vector or a whole matrix. In each row, there should be a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

In this second case, the parameter `increases=TRUE` has to be used.

The procedure uses the resampling method given in the `resamplingMethod` parameter to estimate the p-value of the one-sample test for the mean (denoted further as the one-sample C-test, see Lubiano et al. (2016)). This test checks the null hypothesis that the Aumann-type mean of the fuzzy numbers is equal to a given fuzzy number μ_0 .

Value

This function returns double value which is equal to the p-value of the one-sample C-test.

References

Lubiano, M.A., Montenegro M., Sinova, B., de Saa, S.R., Gil, M.A. (2016) Hypothesis testing for means in connection with fuzzy rating scale-based data: algorithms and applications European Journal of Operational Research, 251, pp. 918-929

See Also

[TwoSampleCTest](#) for the two-sample C-test

Other bootstrapped version of test: [TwoSampleCTest\(\)](#)

Examples

```

# prepare some fuzzy numbers (first type of the initial sample)

fuzzyValues <- matrix(c(0.25,0.5,1,1.25,0.75,1,1.5,2.2,-1,0,0,2),ncol = 4,byrow = TRUE)

# seed PRNG

set.seed(1234)

# calculate the p-value using the classical (i.e. Efron's) bootstrap

OneSampleCTest(fuzzyValues, mu_0 = c(0,0.5,1,1.5))

# calculate the p-value using the VA resampling method

OneSampleCTest(fuzzyValues, mu_0 = c(0,0.5,1,1.5),resamplingMethod = VAmethod)

```

SEResamplingMean

Calculate SE/MSE for the mean of the bootstrapped samples.

Description

SEResamplingMean estimates the standard error (SE) or the mean-squared error (MSE) for the mean while the sample is bootstrapped using one of the applied resampling methods.

Usage

```

SEResamplingMean(
  initialSample,
  resamplingMethod = classicalBootstrap,
  repetitions = 100,
  trueMean = NA,
  theta = 1/3,
  increases = FALSE
)

```

Arguments

initialSample The initial sample which consists of triangular or trapezoidal fuzzy numbers. More than one value can be given in the form of matrix.

resamplingMethod Name of the resampling method, which is used to generate the bootstrapped samples. For the possible names check the values of resamplingMethods vector.

repetitions	Number of the secondary samples which are created using the selected resampling method.
trueMean	If the value is given, then the mean-squared error (MSE) is calculated for this value and the means of the bootstrapped samples. Otherwise, the standard error (SE) is calculated based on the overall mean of the secondary samples.
theta	The weighting parameter for the mid/spread distance applied to calculate the SE/MSE.
increases	If TRUE is used, then the fuzzy numbers should be given in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers should be given in the form: left end of the support, left end of the core, right end of the core, right end of the support.

Details

The input fuzzy values should be triangular or trapezoidal fuzzy numbers, given as a single vector or a whole matrix. In each row, there should be a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

In this second case, the parameter `increases=TRUE` has to be used.

The procedure uses the resampling method given in `resamplingMethod` parameter to create the secondary (bootstrapped) samples. Then the mean (in the form of trapezoidal fuzzy number) is calculated for each new sample. If the parameter `trueMean` is set, then the mean-squared error (MSE) between this true value and each of the sample means is estimated using the Bertoluzza et al.'s (aka mid/spread) distance (with the given weight `theta`, see Bertoluzza et al. (1995)). Otherwise, the overall mean is calculated, and the standard error (SE) is estimated based on the overall mean and each of the sample means. In this case the Bertoluzza et al.'s distance (see Grzegorzewski, Romaniuk (2021)) is also applied.

Value

This function returns list of two double values:

1. mean which is equal to the overall mean (if the SE is calculated) or `trueMean` parameter (if the MSE is calculated),
2. SE which is equal to the estimated SE/MSE of the mean.

The output mean consists of four values which create the trapezoidal fuzzy number.

References

- Bertoluzza, C., Corral, N., Salas, A. (1995) On a new class of distances between fuzzy numbers *Mathware and Soft Computing*, 2 (2), pp. 71-84
- Grzegorzewski, P., Romaniuk, M. (2022) Bootstrap methods for fuzzy data Uncertainty and Imprecision in Decision Making and Decision Support: New Advances, Challenges, and Perspectives, pp. 28-47 Springer

Examples

```

# prepare some fuzzy numbers (first type of the initial sample)

fuzzyValues <- matrix(c(0.25,0.5,1,1.25,0.75,1,1.5,2.2,-1,0,0,2),ncol = 4,byrow = TRUE)

# seed PRNG

set.seed(1234)

# calculate the SE of the mean using the classical (i.e. Efron's) bootstrap

SEResamplingMean(fuzzyValues)

# calculate the SE of the mean using the VA resampling method

SEResamplingMean(fuzzyValues, resamplingMethod = VAMethod)

# calculate the MSE of the given mean using the classical (i.e. Efron's) bootstrap

SEResamplingMean(fuzzyValues, trueMean = c(0,0.5,1,2))

# calculate the MSE of the given mean using the VA resampling method

SEResamplingMean(fuzzyValues, resamplingMethod = VAMethod, trueMean = c(0,0.5,1,2))

```

TwoSampleCTest

Calculate p-value of the two-sample test for the mean

Description

TwoSampleCTest returns the p-value of the two-sample test for the mean using the resampling method.

Usage

```

TwoSampleCTest(
  initialSample1,
  initialSample2,
  numberOfSamples = 100,
  theta = 1/3,
  resamplingMethod = classicalBootstrap,
  increases = FALSE
)

```


Arguments

<code>initialSample1</code>	The first initial sample which consists of triangular or trapezoidal fuzzy numbers. More than one value can be given in the form of matrix.
<code>initialSample2</code>	The second initial sample which consists of triangular or trapezoidal fuzzy numbers. More than one value can be given in the form of matrix.
<code>numberOfSamples</code>	Number of the bootstrapped samples used to estimate the p-value.
<code>theta</code>	The weighting parameter for the mid/spread distance applied in the C-test.
<code>resamplingMethod</code>	Name of the resampling method, which is used to generate the bootstrapped samples. For the possible names check the values of <code>resamplingMethods</code> vector.
<code>increases</code>	If TRUE is used, then the fuzzy numbers should be given in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers should be given in the form: left end of the support, left end of the core, right end of the core, right end of the support.

Details

The input fuzzy values should be triangular or trapezoidal fuzzy numbers, given as a single vector or a whole matrix. In each row, there should be a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

In this second case, the parameter `increases=TRUE` has to be used.

The procedure uses the resampling method given in the `resamplingMethod` parameter to estimate the p-value of the two-sample test for the mean (denoted further as the two-sample C-test, see Lubiano et al. (2016)). This test checks the null hypothesis that the Aumann-type means of two fuzzy samples are equal.

Value

This function returns double value which is equal to the p-value of the two-sample C-test.

References

Lubiano, M.A., Montenegro M., Sinova, B., de Saa, S.R., Gil, M.A. (2016) Hypothesis testing for means in connection with fuzzy rating scale-based data: algorithms and applications *European Journal of Operational Research*, 251, pp. 918-929

See Also

[OneSampleCTest](#) for the one-sample C-test

Other bootstrapped version of test: [OneSampleCTest\(\)](#)

Examples

```
# prepare some fuzzy numbers (first type of the initial sample)

fuzzyValues <- matrix(c(0.25,0.5,1,1.25,0.75,1,1.5,2.2,-1,0,0,2),ncol = 4,byrow = TRUE)

# prepare the slightly shifted second sample

fuzzyValuesShift <- fuzzyValues + 0.5

# seed PRNG

set.seed(1234)

# calculate the p-value using the classical (i.e. Efron's) bootstrap

TwoSampleCTest(fuzzyValues, fuzzyValuesShift)

# calculate the p-value using the VA resampling method

TwoSampleCTest(fuzzyValues, fuzzyValuesShift, resamplingMethod = VAMethod)
```

VAMethod	<i>V(alue)A(mbiguity, left-hand)A(mbiguity, right-hand) resampling method for triangular and trapezoidal fuzzy numbers</i>
----------	--

Description

VAMethod returns the secondary (bootstrapped) sample and uses the resampling scheme which does not change the values, left-hand and right-hand ambiguities of the fuzzy variables from the initial sample (the VAA method, see (Grzegorzewski and Romaniuk, 2022)).

Usage

```
VAMethod(initialSample, b = n, increases = FALSE)
```

Arguments

initialSample	Initial sample of triangular or trapezoidal fuzzy numbers.
b	The number of fuzzy values in the resampled (secondary) sample. If this parameter is not specified, the number of values in the initial sample is used. The parameter should be integer value more than 0.
increases	If TRUE is used, then the initial sample should consist of the fuzzy numbers in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers should be given in the form: left end of the support, left end of the core, right end of the core, right end of the support.

Details

The initial sample should consist of triangular or trapezoidal fuzzy numbers, given as a single vector or a whole matrix. In each row, there should be a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

In this second case, the parameter `increases=TRUE` has to be used.

The resampling procedure produces `b` fuzzy values. During the first step, the fuzzy value from the initial sample is randomly chosen (with repetition). Then the new fuzzy variable, which preserves the value, left- and right-hand ambiguities of the old one, is randomly created. If the parameter `b` is not specified, it is equal to the length of the initial sample. The output is given in the same form as the initial sample.

Value

This function returns matrix with `b` rows of double values. In each row, there is a single resampled fuzzy number. These fuzzy numbers have the same form as the values from the initial sample depending on the provided parameter `increases`.

References

Grzegorzewski, P., Romaniuk, M. (2022) Bootstrap methods for fuzzy data Uncertainty and Imprecision in Decision Making and Decision Support: New Advances, Challenges, and Perspectives, pp. 28-47 Springer

See Also

@seealso [classicalBootstrap](#), [EWmethod](#) for the EW method, [VAFmethod](#) for the VAF method, [dmethod](#) for the d method, [wmethod](#) for the w method

Other resampling functions: [EWmethod\(\)](#), [VAFmethod\(\)](#), [VAMethod\(\)](#), [classicalBootstrap\(\)](#), [dmethod\(\)](#), [wmethod\(\)](#)

Examples

```
# prepare some fuzzy numbers (first type of the initial sample)

fuzzyValues <- matrix(c(0.25,0.5,1,1.25,0.75,1,1.5,2.2,-1,0,0,2),
  ncol = 4,byrow = TRUE)

# generate the secondary sample using the VAA method

set.seed(12345)

VAAmethod(fuzzyValues)

VAAmethod(fuzzyValues,b=4)
```

```
# prepare some fuzzy numbers (second type of the initial sample)

fuzzyValuesInc <- matrix(c(0.25,0.5,1,0.25,0.25,1,1.5,0.7,1,0,0,2),
ncol = 4,byrow = TRUE)

# generate the secondary sample using the VAA method

VAFmethod(fuzzyValuesInc,increases = TRUE)

VAFmethod(fuzzyValuesInc,b=4,increases = TRUE)
```

VAFmethod	<i>V(alue)A(mbiguity)F(uzziness) resampling method for triangular and trapezoidal fuzzy numbers</i>
-----------	---

Description

VAFmethod returns the secondary (bootstrapped) sample and uses the resampling scheme which does not change the values, ambiguities and fuzziness of the fuzzy variables from the initial sample (the VAF method, see (Grzegorzewski et al, 2020)).

Usage

```
VAFmethod(initialSample, b = n, increases = FALSE)
```

Arguments

initialSample	Initial sample of triangular or trapezoidal fuzzy numbers.
b	The number of fuzzy values in the resampled (secondary) sample. If this parameter is not specified, the number of values in the initial sample is used. The parameter should be integer value more than 0.
increases	If TRUE is used, then the initial sample should consist of the fuzzy numbers in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers should be given in the form: left end of the support, left end of the core, right end of the core, right end of the support.

Details

The initial sample should consist of triangular or trapezoidal fuzzy numbers, given as a single vector or a whole matrix. In each row, there should be a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

In this second case, the parameter `increases=TRUE` has to be used.

The resampling procedure produces `b` fuzzy values. During the first step, the fuzzy value from the initial sample is randomly chosen (with repetition). Then the new fuzzy variable, which preserves the value, ambiguity and fuzziness of the old one, is randomly created. If the parameter `b` is not specified, it is equal to the length of the initial sample. The output is given in the same form as the initial sample.

Value

This function returns matrix with `b` rows of double values. In each row, there is a single resampled fuzzy number. These fuzzy numbers have the same form as the values from the initial sample depending on the provided parameter `increases`.

References

Grzegorzewski, P., Hryniewicz, O., Romaniuk, M. (2020) Flexible resampling for fuzzy data based on the canonical representation International Journal of Computational Intelligence Systems, 13 (1), pp. 1650-1662

See Also

[classicalBootstrap](#), [EWmethod](#) for the EW method, [VAFmethod](#) for the VAF method, [VAAmethod](#) for the VAA method, [dmethod](#) for the d method, [wmethod](#) for the w method

Other resampling functions: [EWmethod\(\)](#), [VAAmethod\(\)](#), [VAmethod\(\)](#), [classicalBootstrap\(\)](#), [dmethod\(\)](#), [wmethod\(\)](#)

Examples

```
# prepare some fuzzy numbers (first type of the initial sample)

fuzzyValues <- matrix(c(0.25,0.5,1,1.25,0.75,1,1.5,2.2,-1,0,0,2),
ncol = 4,byrow = TRUE)

# generate the secondary sample using the VAF method

set.seed(12345)

VAFmethod(fuzzyValues)

VAFmethod(fuzzyValues,b=4)

# prepare some fuzzy numbers (second type of the initial sample)

fuzzyValuesInc <- matrix(c(0.25,0.5,1,0.25,0.25,1,1.5,0.7,1,0,0,2),
ncol = 4,byrow = TRUE)

# generate the secondary sample using the VAF method

VAFmethod(fuzzyValuesInc,increases = TRUE)
```

```
VAMethod(fuzzyValuesInc,b=4,increases = TRUE)
```

VAMethod	<i>V(alue)A(mbiguity) resampling method for triangular and trapezoidal fuzzy numbers</i>
----------	--

Description

VAMethod returns the secondary (bootstrapped) sample and uses the resampling scheme which does not change the values and ambiguities of the fuzzy variables from the initial sample (the VA method, see (Grzegorzewski et al, 2020)).

Usage

```
VAMethod(initialSample, b = n, increases = FALSE)
```

Arguments

<code>initialSample</code>	Initial sample of triangular or trapezoidal fuzzy numbers.
<code>b</code>	The number of fuzzy values in the resampled (secondary) sample. If this parameter is not specified, the number of values in the initial sample is used. The parameter should be integer value more than 0.
<code>increases</code>	If TRUE is used, then the initial sample should consist of the fuzzy numbers in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers should be given in the form: left end of the support, left end of the core, right end of the core, right end of the support.

Details

The initial sample should consist of triangular or trapezoidal fuzzy numbers, given as a single vector or a whole matrix. In each row, there should be a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

In this second case, the parameter `increases=TRUE` has to be used.

The resampling procedure produces `b` fuzzy values. During the first step, the fuzzy value from the initial sample is randomly chosen (with repetition). Then the new fuzzy variable, which preserves the value and ambiguity of the old one, is randomly created. If the parameter `b` is not specified, it is equal to the length of the initial sample. The output is given in the same form as the initial sample.

Value

This function returns matrix with `b` rows of double values. In each row, there is a single resampled fuzzy number. These fuzzy numbers have the same form as the values from the initial sample depending on the provided parameter `increases`.

References

Grzegorzewski, P., Hryniewicz, O., Romaniuk, M. (2020) Flexible resampling for fuzzy data based on the canonical representation International Journal of Computational Intelligence Systems, 13 (1), pp. 1650-1662

See Also

@seealso [classicalBootstrap](#), [EWmethod](#) for the EW method, [VAFmethod](#) for the VAF method, [VAAMethod](#) for the VAA method, [dmethod](#) for the d method, [wmethod](#) for the w method

Other resampling functions: [EWmethod\(\)](#), [VAAMethod\(\)](#), [VAFmethod\(\)](#), [classicalBootstrap\(\)](#), [dmethod\(\)](#), [wmethod\(\)](#)

Examples

```
# prepare some fuzzy numbers (first type of the initial sample)

fuzzyValues <- matrix(c(0.25,0.5,1,1.25,0.75,1,1.5,2.2,-1,0,0,2),
  ncol = 4,byrow = TRUE)

# generate the secondary sample using the VA method

set.seed(12345)

VAMethod(fuzzyValues)

VAMethod(fuzzyValues,b=4)

# prepare some fuzzy numbers (second type of the initial sample)

fuzzyValuesInc <- matrix(c(0.25,0.5,1,0.25,0.25,1,1.5,0.7,1,0,0,2),
  ncol = 4,byrow = TRUE)

# generate the secondary sample using the VA method

VAMethod(fuzzyValuesInc,increases = TRUE)

VAMethod(fuzzyValuesInc,b=4,increases = TRUE)
```

wmethod

w method for resampling triangular and trapezoidal fuzzy numbers

Description

wmethod returns the secondary (bootstrapped) sample and uses the resampling scheme based on the special *w density* which is related to the left ends of the cores and increments (i.e. length of the core, left and right increment of the support) of the fuzzy variables from the initial sample (the d method, see (Romaniuk and Hryniewicz, 2019)).

Usage

```
wmethod(initialSample, b = n, increases = FALSE)
```

Arguments

<code>initialSample</code>	Initial sample of triangular or trapezoidal fuzzy numbers.
<code>b</code>	The number of fuzzy values in the resampled (secondary) sample. If this parameter is not specified, the number of values in the initial sample is used. The parameter should be integer value more than 0.
<code>increases</code>	If TRUE is used, then the initial sample should consist of the fuzzy numbers in the form: left increment of the support, left end of the core, right end of the core, right increment of the support. Otherwise, the default value FALSE is used and the fuzzy numbers should be given in the form: left end of the support, left end of the core, right end of the core, right end of the support.

Details

The initial sample should consist of triangular or trapezoidal fuzzy numbers, given as a single vector or a whole matrix. In each row, there should be a single fuzzy number in one of the forms:

1. left end of the support, left end of the core, right end of the core, right end of the support, or
2. left increment of the support, left end of the core, right end of the core, right increment of the support.

In this second case, the parameter `increases=TRUE` has to be used.

The resampling procedure produces `b` fuzzy values. During the first step, the four values are randomly generated using the special *w density*: left end of the core, length of the core, left and right increment of the support. This *w density* is calculated based on the whole fuzzy sample. Then the new fuzzy variable, which preserves the above-mentioned characteristics, is created. If the parameter `b` is not specified, it is equal to the length of the initial sample. The output is given in the same form as the initial sample.

Value

This function returns matrix with `b` rows of double values. In each row, there is a single resampled fuzzy number. These fuzzy numbers have the same form as the values from the initial sample depending on the provided parameter `increases`.

References

Romaniuk, M., Hryniewicz, O. (2019) Interval-based, nonparametric approach for resampling of fuzzy numbers *Soft Computing*, 23 (14), pp. 5883-5903

See Also

@seealso [classicalBootstrap](#), [EWmethod](#) for the EW method, [VAFmethod](#) for the VAF method, [VAAmethod](#) for the VAA method

Other resampling functions: [EWmethod\(\)](#), [VAAmethod\(\)](#), [VAFmethod\(\)](#), [VAMethod\(\)](#), [classicalBootstrap\(\)](#), [dmethod\(\)](#)

Examples

```
# prepare some fuzzy numbers (first type of the initial sample)

fuzzyValues <- matrix(c(0.25,0.5,1,1.25,0.75,1,1.5,2.2,-1,0,0,2),
  ncol = 4,byrow = TRUE)

# generate the secondary sample using the w method

set.seed(12345)

wmethod(fuzzyValues)

wmethod(fuzzyValues,b=4)

# prepare some fuzzy numbers (second type of the initial sample)

fuzzyValuesInc <- matrix(c(0.25,0.5,1,0.25,0.25,1,1.5,0.7,1,0,0,2),
  ncol = 4,byrow = TRUE)

# generate the secondary sample using the w method

wmethod(fuzzyValuesInc,increases = TRUE)

wmethod(fuzzyValuesInc,b=4,increases = TRUE)
```

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